XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

A SCHEME TO SOLVE NON-LINEAR CONDUCTION PHENOMENA BASED ON A SEQUENCE OF LINEAR PROBLEMS

Sérgio Frey
Grupo de Estudos Térmicos e Energéticos (GESTE)
Departamento de Engenharia Mecânica - Universidade Federal do Rio Grande do Sul
Rua Sarmento Leite, 425 – 90050-170 Porto Alegre/RS, Brazil
E-mail: frey@mecanica.ufrgs.br - http://www.mecanica.ufrgs.br/prof/frey/frey.htm
Maria Laura Martins-Costa
Laboratório de Mecânica Teórica e Aplicada (LMTA)
Departamento de Engenharia Mecânica - Universidade Federal Fluminense
Rua Passo da Pátria, 156 – 24210-240 Niterói/RJ, Brazil
E-mail: laura@caa.uff.br
Rogério M. Saldanha da Gama
Laboratório Nacional de Computação Científica (LNCC/CNPq)
Rua Getúlio Vargas, 333 – 25651-070 Petrópolis/RJ, Brazil
E-mail: rsgama@domain.com.br

Abstract. *The present work proposes a scheme to solve non linear heat conduction problems based on a sequence of linear problems. An opaque three-dimensional plate with a non-linear temperature-dependent source dominating the conductive operator is studied by employing a convergent sequence of linear problems whose simulation technique, based on a finite element methodology, is known. Since the classical Galerkin method presents instabilities when subjected to very high source-dominated regimen, a Gradient-Galerkin/Least-Squares formulation is presented for a linear problem - an element of the convergent sequence.*

***Keywords*:** *Three-dimensional heat conduction, Non-linear heat source, Variational principle, Finite element method, Stabilized methods.*

1. INTRODUCTION

In most cases, an accurate mathematical modelling of real problems involving transport phenomena gives rise to nonlinear systems of partial differential equations. Numerical strategies to deal with these problems, such as finite element and finite difference methods, after performing a convenient discretization lead to algebraic systems of equations. A great variety of numerical solution methods for linear systems is available but if the system is a nonlinear one (unfortunately the majority of cases) there is neither guarantee of existence nor uniqueness of solution. Some numerical schemes to solve these problems (e.g. quasi-Newton methods) are available in the literature. A distinct procedure is proposed in the present work: a new mathematical description for a large class of nonlinear heat conduction problems which are treated as a convergent sequence of continuous linear problems. A variational formulation is then introduced, since each element of the sequence is a function that minimizes a convex quadratic and coercive functional. A numerical discretization, such as finite elements, may be used to treat each linear problem.

Finite element methods are usually based on Galerkin approximation, originally introduced for structural problems which, assuming some restrictions usually present in engineering practical applications (Franca et al., 1992 and references therein), gives rise to symmetric elliptic operators and generates rather optimum convergence rates. However, numerical pathologies in the Galerkin approximations such as the locking of the velocity field and spurious oscillations in the pressure field may occur if Galerkin method is applied to fluid problems. These undesirable pathologies may be present even in such kinematical cases when a temperature-dependent heat source dominates the classical diffusive operator.

The present work studies the heat transfer process in an opaque three-dimensional plate (Saldanha da Gama, 1999) with a non-linear temperature-dependent source dominating the conductive operator. The adopted mechanical model is obtained assuming the existence of a heat transfer from/to the plate following Newton's law of cooling. Besides, an integration of this model on plate thickness direction produces a two dimensional model in terms of a mean plate temperature. The resulting non-linear conduction problem is treated as the limit of a convergent sequence of linear problems. A variational formulation is then proposed, since each linear problem of the above mentioned sequence has an equivalent minimum principle. A finite element strategy is then employed to approximate each linear element of the above mentioned sequence. Since numerical simulations have attested the instability inherent to Galerkin formulation in the presence of very high source-dominated regimen (Frey et al., 1998), a stabilized formulation based upon Gradient-Galerkin/Least-Squares, from now on referred as GGLS methodology (Franca & Dutra do Carmo, 1989), which is able to generate stable approximations even for very high source-dominated regimen (Hughes, 1987), is presented (Frey et al., 1999). The technique employed allows to perform complex simulations by means of well stabilished tools, even for problems presenting spurious oscilations, arising from dominant non linear temperature dependent heat sources.

2. MECHANICAL MODELLING

Let us consider a body represented by the bounded open set $\mathcal{B}$ defined by

$$\mathcal{B} = \{(x, y, z) \in \mathbf{R}^3 \quad \text{such that } (x, y) \in \Omega \subset \mathbf{R}^2, -L < z < L\} \tag{1}$$

in which Ω is a bounded open set of $\mathbf{R}^2$ and L is a positive constant. The set $\mathcal{B}$ is called a plate (see Fig.1) and, for for small values of L, the heat transfer process in the plate $\mathcal{B}$ is conveniently described taking into account its geometrical features. In other words, the energy transfer phenomenon in $\mathcal{B}$ may be described, in a certain sense, in the set Ω (Saldanha da Gama, 1997). In this work, we will treat the heat transfer process in a thin plate originally represented by:

$$
\begin{aligned}
\nabla \cdot (\kappa \nabla T) + \dot{q} &= 0 && \text{in } \mathcal{B} \\
T &= T_g && \text{on } \Gamma_g \\
\kappa \nabla T \cdot \mathbf{n}_{\partial\Omega} &= 0 && \text{on } \Gamma_h \\
-\kappa \nabla T \cdot \mathbf{e}_z &= \hbar_+ (T - T_+) && \text{for } z = +L \\
\kappa \nabla T \cdot \mathbf{e}_z &= \hbar_- (T - T_-) && \text{for } z = -L
\end{aligned}
\tag{2}
$$

in which T is the temperature field, Γ_g is the region of the boundary $\partial\Omega$ on which essential (Dirichlet) conditions are imposed while Γ_h is subjected to the natural (Neumann) ones, $\mathbf{n}_{\partial\Omega}$ is the unit outward normal to $\partial\Omega$, $\kappa(x, y) > 0$ the thermal conductivity, $\hbar_+(x, y)$ and $\hbar_-(x, y)$ convection heat transfer coefficients, $T_+(x, y)$ and $T_-(x, y)$ reference temperatures and the energy supply $\dot{q}$ is supposed to be a non-linear and non-increasing function of the temperature, given by $\dot{q} = \hat{q}(T)$.

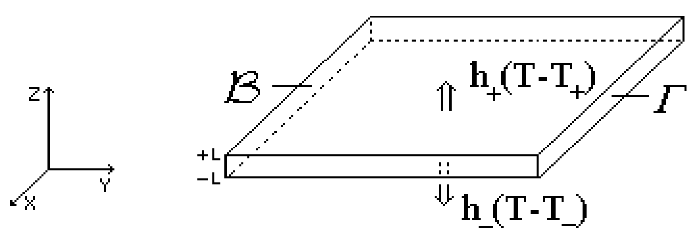

Figure 1 - Energy transfer in a three-dimensional plane plate.

2.1 The plate approximation

Integrating the first equation of (2) over the thickness $2L$ and using the boundary conditions stated in (2), we obtain

$$
\begin{aligned}
\int_{-L}^{+L} \{\nabla \cdot (\kappa \nabla T) + \dot{q}\} dz - [\hbar_+ (T - T_+)]_{z=+L} & \\
-[\hbar_- (T - T_-)]_{z=-L} &= 0 \quad \text{in } \Omega \\
T &= T_g \quad \text{on } \Gamma_g \\
\kappa \nabla T \cdot \mathbf{n}_{\partial\Omega} &= 0 \quad \text{on } \Gamma_h
\end{aligned}
\tag{3}
$$

At this point it is convenient to define a mean temperature evaluated over the thickness of the plate, $\theta = \frac{1}{2L} \int_{-L}^{+L} T dz$. Assuming that $\dot{q} \in L^2(\mathcal{B})$,

$$
L^2(\mathcal{B}) = \left\{ q \mid \int_{\mathcal{B}} q^2 d\Omega < \infty \right\} \tag{4}
$$

and that $\mathcal{B}$ has the cone property (Maz'ja, 1995), both T and θ are continuous and bounded functions. Thus the approximation $\theta \cong T$ in $\mathcal{B}$ may be considered for a thin plate (very small L) and problem (3) gives rise to,

$$\begin{aligned} \nabla \cdot (\kappa \nabla \theta) + \dot{q} - \frac{1}{2L}[\hbar_+(\theta - T_+) \\ +\hbar_-(\theta - T_-)] = 0 \quad \text{in } \Omega \\ \theta = \theta_g \quad \text{on } \mathit{\Gamma}_g \\ \kappa \nabla \theta \cdot \mathbf{n}_{\partial\Omega} = 0 \quad \text{on } \mathit{\Gamma}_h \end{aligned} \tag{5}$$

in which the unknown θ depends only on (x, y). Considering the energy supply $\dot{q}$, problem (5) may be expressed as

$$\begin{aligned} -\nabla \cdot (\kappa \nabla \theta) + r + \mu\theta = \lambda \quad \text{in } \Omega \\ \theta = \theta_g \quad \text{on } \mathit{\Gamma}_g \\ \kappa \nabla \theta \cdot \mathbf{n}_{\partial\Omega} = 0 \quad \text{on } \mathit{\Gamma}_h \end{aligned} \tag{6}$$

where $\lambda(x, y)$ and $\mu(x, y)$ are known functions defined by

$$\lambda = \frac{1}{2L}(\hbar_+ T_+ + \hbar_- T_-) \quad ; \quad \mu = \frac{1}{2L}(\hbar_+ + \hbar_-) \tag{7}$$

3. SOLUTION CONSTRUCTION

A technique enabling to represent the non-linear conduction problem presented in Eqs.(6)-(7) as the limit of a convergent sequence of linear problems is now proposed. Each element of the sequence, which may be obtained from the minimization of a convex quadratic and coercive functional, represents a three-dimensional plate with temperature dependent heat source.

3.1 On the convergence of process

Assuming the thermal condutivity κ as a constant, the problem represented by equations (6)-(7) may be rewritten as:

$$\begin{aligned} \Delta\theta + f(\theta) = 0 \quad \text{in } \Omega \\ \theta = \theta_g \geq 0 \quad \text{on } \mathit{\Gamma}_g \\ \nabla\theta \cdot \mathbf{n}_{\partial\Omega} = 0 \quad \text{on } \mathit{\Gamma}_h \end{aligned} \tag{8}$$

where $f(\theta)$ is a nonlinear, nonnegative and decreasing function of θ given by

$$f(\theta) = -\frac{r + \mu\theta - \lambda}{\kappa} \tag{9}$$

Proposition. A proposed solution θ of the above stated problem is given by the limit of the sequence $[\theta_1, \theta_2, \ldots, \theta_i, \ldots]$ with $\theta_1 \equiv 0$, whose elements are given by

$$
\begin{aligned}
\Delta\theta_{i+1} + f(\theta_i) - \alpha(\theta_{i+1} - \theta_i) = 0 \quad & \text{in} \quad \Omega \\
\theta_{i+1} = \theta_g \quad & \text{on } \Gamma_g \\
\nabla\theta_{i+1} \cdot \mathbf{n}_{\partial\Omega} = 0 \quad & \text{on } \Gamma_h
\end{aligned}
\tag{10}
$$

where α is a sufficiently large constant.

Proof. Problem (10) may be rewiritten as

$$
\begin{aligned}
\Delta(\theta_{i+1} - \theta_i) - [\alpha(\theta_{i+1} - \theta_i) - \alpha(\theta_i - \theta_{i-1})] + f(\theta_i) - f(\theta_{i-1}) = 0 \quad & \text{in} \quad \Omega \\
\theta_{i+1} - \theta_i = 0 \quad & \text{on } \Gamma_g \\
\nabla(\theta_{i+1} - \theta_i) \cdot \mathbf{n}_{\partial\Omega} = 0 \quad & \text{on } \Gamma_h
\end{aligned}
\tag{11}
$$

Since $\theta_1 \equiv 0$, $f(\theta)$ is nonnegative and $\theta_2 \geq 0$, it may be concluded that $\theta_2 \geq \theta_1$ everywhere. Besides, once that in the region where $\theta_{i+1} - \theta_i$ reaches its minimum one must have

$$
\alpha(\theta_{i+1} - \theta_i) \geq [\alpha(\theta_i - \theta_{i-1}) + f(\theta_i) - f(\theta_{i-1})] \tag{12}
$$

and since α must be sufficiently large, it comes that $\theta_{i+1} - \theta_i \geq 0$ everywhere, leading to the conclusion that

$$
\theta_1 \leq \theta_2 \leq \theta_3 \ldots \tag{13}
$$

which is equivalent to say that the sequence $[\theta_1, \theta_2, \ldots, \theta_i, \ldots]$ is a non-decreasing one.

In order to show that the non-decreasing sequence $[\theta_1, \theta_2, \ldots, theta_i, \ldots]$ converges one must show that it has an upper bound. If θ is the unique solution of the problem, it may be stated as

$$
\begin{aligned}
\Delta(\theta - \theta_{i+1}) + \alpha(\theta_{i+1} - \theta_i) + f(\theta) - f(\theta_i) = 0 \quad & \text{in} \quad \Omega \\
\theta - \theta_{i+1} = 0 \quad & \text{on } \Gamma_g \\
\nabla(\theta - \theta_{i+1}) \cdot \mathbf{n}_{\partial\Omega} = 0 \quad & \text{on } \Gamma_h
\end{aligned}
\tag{14}
$$

in order that $(\theta - \theta_{i+1})$ reaches the minimum it is necessary that

$$
\alpha(\theta_{i+1} - \theta_i) \leq f(\theta_i) - f(\theta) \tag{15}
$$

Finally, since it has been assumed that the function f is a decreasing function of θ and $\theta_{i+1} \geq \theta_i$ one may conclude that

$$
\theta \geq \theta_i \tag{16}
$$

ensuring the convergence of the sequence to the solution of the problem.

Obtaining each $\boldsymbol{\theta_i}$. Assuming that the the solution θ of the sequence $[\theta_1, \theta_2, \ldots, \theta_i, \ldots]$ is given by

$$\theta = \lim_{i \to \infty} \theta_i \tag{17}$$

a variational formulation may be constructed by making each θ_i the field which minimizes the functional $I_{i+1}[\phi]$ defined as below,

$$I_{i+1}[\phi] = \int_\Omega \left\{ \frac{\kappa}{2}(|\nabla \phi|^2) - \overline{\lambda}_i \phi + \frac{\overline{\mu}_i}{2} |\phi|^2 \right\} d\Omega \tag{18}$$

in which ϕ is any admissible field $\phi \in W$,

$$W = \{\phi \in L^2(\Omega) \,|\, \partial_{x_i}\phi \in L^2(\Omega), \text{ for } i = 1, 2\,, \phi = \theta_g \text{ on } \mathit{\Gamma}_g\} \tag{19}$$

and $\overline{\lambda}_i$ and $\overline{\mu}_i$ are given by

$$\begin{aligned} \overline{\lambda}_i &= -\lambda + F(\theta_i) - \alpha\theta_i \\ \overline{\mu}_i &= \mu + \alpha \end{aligned} \tag{20}$$

The functional $I_{i+1}[\phi]$ is quadratic, convex and coercive (Berger, 1977; Saldanha da Gama, 1997), since κ and $\overline{\mu}_i$ are positive valued functions. In other words, this functional admits only one minimum which is reached when $\phi \cong \theta_i$, being θ_i the solution of Eq.(10), the Euler-Lagrange equation of functional (18).

After performing the minimization of each functional $I_{i+1}[\phi]$, the coefficients $\overline{\lambda}_i$ and $\overline{\mu}_i$ must be recalculated. This operation is repeated until convergence is achieved.

4. AN APPLICATION TO A LINEAR PROBLEM

4.1 Finite element discretization of a linear problem

In this section a finite element method is employed to approximate each element of the convergent sequence of linear problems which represents the energy transfer in thin plates subjected to nonlinear temperature-dependent heat sources (Eqs.(6)-(7)). The problems considered herein are defined on a bounded domain $\Omega \subset \mathbf{R}^2$, with a boundary $\mathit{\Gamma}$,

$$\begin{cases} \mathit{\Gamma} = \overline{\mathit{\Gamma}}_g \cup \overline{\mathit{\Gamma}}_h\,, \\ \mathit{\Gamma}_g \cap \mathit{\Gamma}_h = \emptyset\,,\ \mathit{\Gamma}_g \neq 0 \end{cases} \tag{21}$$

with $\mathit{\Gamma}_g$ and $\mathit{\Gamma}_h$ defined as in Eq.(2). A partition $\mathcal{C}_h$ of domain $\overline{\Omega}$ into elements K consisting of convex quadrilaterals is performed in the usual way (Ciarlet, 1978),

$$\begin{cases} \overline{\Omega} = \displaystyle\bigcup_{K \in \mathcal{C}_h} \overline{\Omega}_K \\ \Omega_{K_1} \cap \Omega_{K_2} = \emptyset\,, \qquad \forall K_1, K_2 \in \mathcal{C}_h \end{cases} \tag{22}$$

and the finite element subspaces for temperature field W^h are defined by,

$$W^h = \{\phi \in H^1(\Omega) \,|\, \phi_{|K} \in P_k(K),\ \forall K \in \mathcal{C}_h\,, \phi = 0 \text{ on } \mathit{\Gamma}_g\} \tag{23}$$

$$W_g^h = \{\phi \in H^1(\Omega) \mid \phi_{|K} \in P_k(K),\ \forall K \in \mathcal{C}_h\,,\ \ \phi = \theta_g \text{ on } \Gamma_g\} \tag{24}$$

where $P_k(K)$ denotes the space of polynomials of degree k greater than zero and $H^1(\Omega)$ is the Sobolev space of functions with square-integrable value and $1^{\underline{st}}$−derivatives in Ω - see Adams (1975),

$$H^1(\Omega) = \left\{ v \in L^2(\Omega) \mid \frac{\partial v}{\partial x_i} \in L^2(\Omega),\ \text{for } i = 1, 2 \right\} \tag{25}$$

Based upon the approximated subsets described by Eqs.(23)-(24) and supposing that the finite element approximation θ_{i+1}^h admits the representation (Hughes, 1987)

$$\theta_{i+1}^h(\mathbf{x}) = \varphi^h(\mathbf{x}) + \theta_g^h(\mathbf{x})\,, \tag{26}$$

where $\varphi^h \in W^h$ and $\theta_g^h = \theta_g$ on Γ_h, we can construct a Gradient-Galerkin/Least-Squares (GGLS) formulation - introduced by Franca & Dutra do Carmo (1989), which adds to Galerkin formulation a least-squares form of the gradient of the Euler-Lagrage equations in order to enhance stability on the H^1-seminorm - to each linear problem in the convergent sequence representing Eqs.(6)-(7) as: *Given* $r\colon \overline{\Omega} \to \mathbf{R}$ *and* $\theta_g\colon \overline{\Gamma}_g \to \mathbf{R}$, *Find* $\theta_{i+1}^h \in W_g^h$, $\theta_{i+1}^h = \varphi^h + \theta_g^h$ *with* $\varphi^h \in W^h$, *such that*

$$\begin{aligned}
&\int_\Omega \kappa \nabla\varphi^h \cdot \nabla\phi^h d\Omega + \int_\Omega \overline{\mu}_i \varphi^h \phi^h d\Omega \\
&+ \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} (\overline{\mu}_i\varphi^h - \nabla\cdot(\kappa\nabla\varphi^h))\,\tau(\overline{\mu}_i\phi^h - \nabla\cdot(\kappa\nabla\phi^h))d\Omega \\
&+ \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} (\nabla(\overline{\mu}_i\varphi^h) - \nabla(\nabla\cdot(\kappa\nabla\varphi^h)))\cdot\gamma(\nabla(\overline{\mu}_i\phi^h) - \nabla(\nabla\cdot(\kappa\nabla\phi^h)))d\Omega \\
&= \int_\Omega \overline{\lambda}_i \phi^h d\Omega - \int_\Omega \kappa\nabla\theta_g^h \cdot \nabla\phi^h d\Omega - \int_\Omega \overline{\mu}_i \theta_g^h \phi^h d\Omega \\
&+ \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} \overline{\lambda}_i\,\tau(\overline{\mu}_i\phi^h - \nabla\cdot(\kappa\nabla\phi^h))d\Omega \\
&+ \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} \nabla\overline{\lambda}_i \cdot \gamma(\nabla(\overline{\mu}_i\phi^h) - \nabla(\nabla\cdot(\kappa\nabla\phi^h)))d\Omega \\
&- \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} (\overline{\mu}_i\theta_g^h - \nabla\cdot(\kappa\nabla\theta_g^h))\,\tau(\overline{\mu}_i\phi^h - \nabla\cdot(\kappa\nabla\phi^h))d\Omega \\
&- \sum_{K\in\mathcal{C}_h} \int_{\Omega_K} (\nabla(\overline{\mu}_i\theta_g^h) - \nabla(\nabla\cdot(\kappa\nabla\theta_g^h)))\cdot\gamma(\nabla(\overline{\mu}_i\phi^h) - \nabla(\nabla\cdot(\kappa\nabla\phi^h)))d\Omega \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \forall\phi^h \in W^h
\end{aligned} \tag{27}$$

where the stability parameter τ and γ are defined from error analysis (Valentin & Franca, 1995) considerations as follows:

$$\tau(\mathbf{x}, \omega_K(\mathbf{x})) = \frac{1}{2\overline{\mu}_i}\xi(\omega_K(\mathbf{x})) \tag{28}$$

$$\gamma(\mathbf{x}, \omega_K(\mathbf{x})) = \frac{h_K^2}{4\overline{\mu}_i}\xi(\omega_K(\mathbf{x})) \tag{29}$$

$$\omega_K(\mathbf{x}) = \frac{m_k \overline{\mu}_i h_K^2}{2\kappa(\mathbf{x})} \tag{30}$$

$$\xi(\omega_K(\mathbf{x})) = \begin{cases} \omega_K(\mathbf{x}) & , 0 \leq \omega_K(\mathbf{x}) < 1 \\ 1 & , \omega_K(\mathbf{x}) > 1 \end{cases} \tag{31}$$

$$m_k = \min\left\{\frac{1}{3},\ 2C_k\right\} \tag{32}$$

$$C_k \sum_{K \in \mathcal{C}_h} h_K^2 \|\Delta\phi\|_{0,K}^2 \leq \|\nabla\phi\|_0^2 \qquad \phi \in W_h \tag{33}$$

Remarks

1. Glancing through the GGLS formulation defined by Eqs.(27), we may note that whether the stability parameters τ and γ are taken to be zero, Galerkin formulation would be recovered for Eqs.(6)-(7).
2. In Frey et al. (1999) the stability parameters are presented, based on the error analysis obtained for the GGLS method for an advective-diffusive model with temperature-dependent source by Valentin & Franca (1995), derived for the particular case of of absence of advective velocity field. Since each element of the convergent sequence that represents the conduction in a plate with a nonlinear source studied in the present work is a linear problem, like the one considered by Frey et al. (1999), the same stability parameters evaluation holds for the present case.

4.2 Results for each linear problem

In this section we present some two-dimensional simulations of the heat transfer process in a rectangular plate subjected to a thermal dominant heat source. The geometry and boundary conditions of the problem are sketched in Fig.2a: the domain is a biunity plane plate subjected to both Dirichlet and Neumann boundary conditions,

$$\theta = 0 \begin{cases} x = -0.5 & , -0.5 \leq y \leq +0.5; \\ y = -0.5 & , -0.5 \leq x \leq +0.5. \end{cases} \tag{34}$$

$$\text{(a) } \theta = 1 \text{ or (b) } \nabla\theta \cdot \mathbf{n}_{\partial\Omega} = 0 \\ \begin{cases} x = +0.5 & , -0.5 \leq y \leq +0.5; \\ y = +0.5 & , -0.5 \leq x \leq +0.5. \end{cases} \tag{35}$$

the diffusivity is $\kappa = 10^{-8}$, while the thermal source λ and the coefficient μ of the zero-derivative term of eq.(7) were taken equal to one. The computational domain Ω was discretized by a uniform mesh consisting of 900 bilinear elements with 961 degrees-of-freedom.

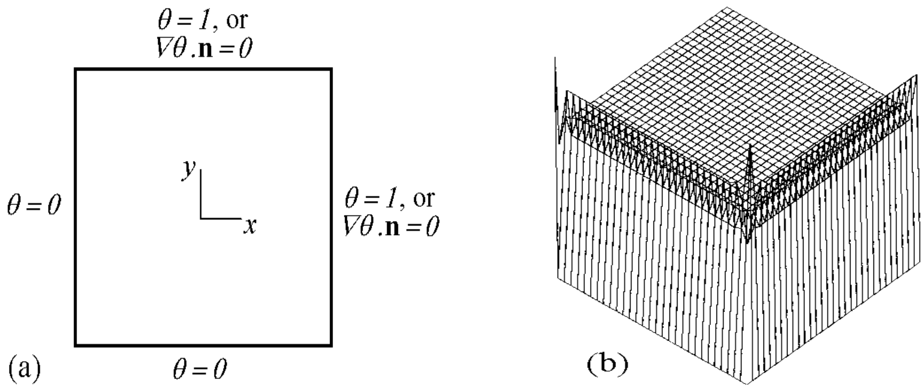

Figure 2 - (a) Problem statement. (b) Galerkin formulation - Dirichlet conditions.

The Galerkin approximation of Eqs.(6)-(7) subjected to Dirichlet boundary conditions, when the diffusive term is largely dominated by the temperature-dependent source term - in which the latter is 10^8 times greater than the former - is shown in Fig. 2b. In this figure Galerkin spurious oscillation for a very high zero-order-dominated situation were observed and neither an alternative Neumann boundary condition nor a higher order interpolation nor a mesh refinement were able to deal with these oscillations (Frey et al., 1998).

The results of the GGLS formulation defined by Eqs.(27)-(33) are presented in Fig.3 once again for very high source-dominated situations ($\kappa = 10^{-8}$ and $\lambda = \mu = 1$). The same mesh discretization of the Galerkin simulation has been employed in this stabilized simulation of Eqs.(6)-(7), subjected to both Dirichlet boundary conditions (Fig. 3a) and Neumann boundary conditions (Fig. 3b).

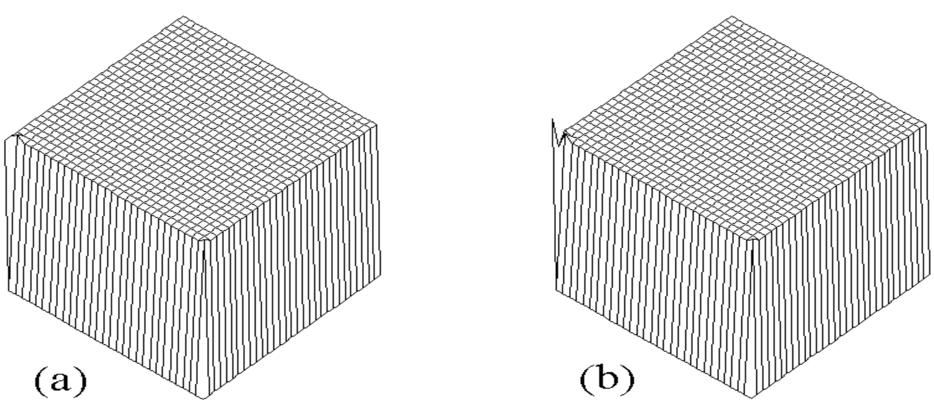

Figure 3 - GGLS formulation (a) Dirichlet conditions (b) Neumann conditions.

As it can be seen from Fig.3 and from all situations considered in Frey et al. (1999), the GGLS formulation was able to deal precisely with very high zero-order-dominated situations and its results are similar to the homogeneous Dirichlet problem computed in Franca and Dutra do Carmo (1989).

5. FINAL REMARKS

In this article a scheme to solve a conduction heat transfer process in an opaque three-dimensional plate with non-linear temperature-dependent heat source was proposed. The non-linear resulting problem was regarded as the limit of a sequence whose convergence has been proven and whose elements were obtained from the minimization of a convex quadatic functional. In short, a nonlinear problem has been treated as a convergent sequence of linear problems. Such a technique allows to carry out complex simulations by means of well stabilished numerical tools, such as GGLS finite element approximations, even for problems with dominant non linear temperature-dependent heat source - which may lead to spurious oscilations in a classical Galerkin approach.

Acknowledgements

The authors S. Frey and M.L. Martins-Costa gratefully acknowledge the finantial support provided by the agency CNPq through grants n^o 350747/93-8 and 300404/91-3.

REFERENCES

Adams, R.A., 1975, Sobolev Spaces, Academic Press, New York.

Berger, M.S., 1977, Nonlinearity and Functional Analysis, Academic Press, London.

Ciarlet, P.G., 1978, The Finite Element Method for Elliptic Problems, North-Holland, Amsterdam.

Franca, L.P. & Dutra do Carmo, E.G., 1989, The Galerkin-Gradient-Least-Squares Method, Comput. Methods Appl. Mech. Engrg. vol. 74, pp.41-54.

Franca, L.P., Frey, S. and Hughes, T.J.R., 1992, Stabilized Finite Element Methods: I. Application to the Advective-Diffusive Model, Comput. Methods App. Mech. Engrg. vol. 95, pp. 253-276.

Frey, S., Martins-Costa, M.L. and Saldanha da Gama, R.M., 1998, Classical Galerkin Shortcomings on Steady Heat Transfer Conduction Simulation in a Plate with Thermal Source, Mech. Research Comm., vol. 25, n.5, pp. 561-567.

Frey, S., Martins-Costa, M.L. and Saldanha da Gama, R.M., 1999, Stabilized Finite Element Approximation for Heat Conduction in a 3-D Plate with Dominant Thermal Source, to appear in Comput. Mech..

Hughes, T.J.R., 1987, The Finite Element Method: Linear Static and Dynamic Finite Element Analysis, Prentice-Hall, Singapore.

Maz'ja, V.G., 1995, Sobolev Spaces, Springer-Verlag, Berlin.

Saldanha da Gama, R.M., 1997, Mathematical Modelling of the Non-Linear Heat Transfer Process in a Great Shell Surrounded by a Nonparticipating Medium, Int. J. Non-Linear Mech., vol. 32, n. 5, pp. 885-904.

Saldanha da Gama, R.M., 1999, A Finite Element Approximation for the Heat Transfer Process in a Plate with a Nonuniform Temperature-Dependent Source, to appear in J. Braz. Soc. Mech. Sci..

Valentin, F.G.C. & Franca, L.P., 1995, Combining Stabilized Finite Element Methods, Comp. App. Math., vol. 14, n. 3, pp. 285-300.